Даны координаты пирамиды: A1(-5,0,1), A2(-4,-2,3), A3(6,2,11), A4(3,4,9)  
**1) Координаты векторов**.  
Координаты векторов находим по формуле:  
X = xj - xi; Y = yj - yi; Z = zj - zi  
здесь X,Y,Z координаты вектора; xi, yi, zi - координаты точки Аi; xj, yj, zj - координаты точки Аj;  
Например, для вектора A1A2  
X = x2 - x1; Y = y2 - y1; Z = z2 - z1  
X = -4-(-5); Y = -2-0; Z = 3-1  
A1A2(1;-2;2)  
A1A3(11;2;10)  
A1A4(8;4;8)  
A2A3(10;4;8)  
A2A4(7;6;6)  
A3A4(-3;2;-2)  
**2) Модули векторов** (длина ребер пирамиды)  
Длина вектора a(X;Y;Z) выражается через его координаты формулой:  
https://chart.googleapis.com/chart?cht=tx&chl=|a|%20=%20\sqrt%7bX%5e%7b2%7d%20%2B%20Y%5e%7b2%7d%20%2B%20Z%5e%7b2%7d%7d  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b1%7dA_%7b2%7d|%20=%20\sqrt%7b1%5e%7b2%7d%20%2B%202%5e%7b2%7d%20%2B%202%5e%7b2%7d%7d%20=%20\sqrt%7b9%7d%20=%203  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b1%7dA_%7b3%7d|%20=%20\sqrt%7b11%5e%7b2%7d%20%2B%202%5e%7b2%7d%20%2B%2010%5e%7b2%7d%7d%20=%20\sqrt%7b225%7d%20=%2015  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b1%7dA_%7b4%7d|%20=%20\sqrt%7b8%5e%7b2%7d%20%2B%204%5e%7b2%7d%20%2B%208%5e%7b2%7d%7d%20=%20\sqrt%7b144%7d%20=%2012  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b2%7dA_%7b3%7d|%20=%20\sqrt%7b10%5e%7b2%7d%20%2B%204%5e%7b2%7d%20%2B%208%5e%7b2%7d%7d%20=%20\sqrt%7b180%7d%20=%2013.416  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b2%7dA_%7b4%7d|%20=%20\sqrt%7b7%5e%7b2%7d%20%2B%206%5e%7b2%7d%20%2B%206%5e%7b2%7d%7d%20=%20\sqrt%7b121%7d%20=%2011  
https://chart.googleapis.com/chart?cht=tx&chl=|A_%7b3%7dA_%7b4%7d|%20=%20\sqrt%7b3%5e%7b2%7d%20%2B%202%5e%7b2%7d%20%2B%202%5e%7b2%7d%7d%20=%20\sqrt%7b17%7d%20=%204.123  
**3) Угол между ребрами**.  
Угол между векторами a1(X1;Y1;Z1), a2(X2;Y2;Z2) можно найти по формуле:  
https://chart.googleapis.com/chart?cht=tx&chl=cos%20\gamma%20%20%20=%20\frac%7ba_%7b1%7da_%7b2%7d%7d%7b|a_%7b1%7d|\cdot%20|a_%7b2%7d|%7d  
где a1a2 = X1X2 + Y1Y2 + Z1Z2  
Найдем угол между ребрами A1A2(1;-2;2) и A1A3(11;2;10):  
https://chart.googleapis.com/chart?cht=tx&chl=cos%20\gamma%20%20%20=%20\frac%7b1\cdot%2011%20%2B%20(-2)\cdot%202%20%2B%202\cdot%2010%7d%7b3\cdot%2015%7d%20=%200.6  
γ = arccos(0.6) = 53.1320  
**7) Проекция вектора**  
Проекцию вектора *b* на вектор *a* можно найти по формуле:  
https://chart.googleapis.com/chart?cht=tx&chl=Pp_%7ba%7db%20=%20\frac%7ba%20b%7d%7b|a|%7d  
Найдем проекцию вектора A3A4 на вектор A1A2  
https://chart.googleapis.com/chart?cht=tx&chl=Pp_%7bA_%7b1%7dA_%7b2%7d%7dA_%7b3%7dA_%7b4%7d%20=%20\frac%7b(-3)\cdot%201%20%2B%202(-2)%20%2B%20(-2)\cdot%202%7d%7b\sqrt%7b9%7d%7d%20=%209

**4) Площадь грани**  
Площадь грани можно найти по формуле:  
https://chart.googleapis.com/chart?cht=tx&chl=S%20=%20\frac%7b1%7d%7b2%7d%20|a|\cdot%20|b|%20sin%20\gamma  
где  
https://chart.googleapis.com/chart?cht=tx&chl=sin%20\gamma%20%20=%20\sqrt%7b1%20-%20cos%20\gamma%5e%7b2%7d%7d  
Найдем площадь грани A1A2A3  
Найдем угол между ребрами A1A2(1;-2;2) и A1A3(11;2;10):  
https://chart.googleapis.com/chart?cht=tx&chl=cos%20\gamma%20%20%20=%20\frac%7b1\cdot%2011%20%2B%20(-2)\cdot%202%20%2B%202\cdot%2010%7d%7b3\cdot%2015%7d%20=%200.6  
https://chart.googleapis.com/chart?cht=tx&chl=sin%20\gamma%20%20=%20\sqrt%7b1%20-%200.6%5e%7b2%7d%7d%20=%200.8  
Площадь грани A1A2A3  
  
Найдем площадь грани с учётом геометрического смысла векторного произведения:  
https://chart.googleapis.com/chart?cht=tx&chl=S%20=%20\frac%7b1%7d%7b2%7d%20|\overline%7bA_%7b1%7dA_%7b2%7d%7d%20x%20\overline%7bA_%7b1%7dA_%7b3%7d%7d|  
Векторное произведение:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | i | j | k | | 1 | -2 | 2 | | 11 | 2 | 10 | |  | | = |

= i((-2) • 10-2 • 2) - j(1 • 10-11 • 2) + k(1 • 2-11 • (-2)) = -24i + 12j + 24k

**5) Объем пирамиды**.  
Объем пирамиды, построенный на векторах a1(X1;Y1;Z1), a2(X2;Y2;Z2), a3(X3;Y3;Z3) равен:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=V%20=%20\frac%7b1%7d%7b6%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | X1 | Y1 | Z1 | | X2 | Y2 | Z2 | | X3 | Y3 | Z3 | |  | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| https://chart.googleapis.com/chart?cht=tx&chl=V%20=%20\frac%7b1%7d%7b6%7d | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | 1 | -2 | 2 | | 11 | 2 | 10 | | 8 | 4 | 8 | |  | | https://chart.googleapis.com/chart?cht=tx&chl=%20=%20\frac%7b48%7d%7b6%7d%20=%208 |

Находим определитель матрицы  
∆ = 1 • (2 • 8-4 • 10)-11 • ((-2) • 8-4 • 2)+8 • ((-2) • 10-2 • 2) = 48  
**8) Уравнение плоскости**.  
Если точки A1(x1; y1; z1), A2(x2; y2; z2), A3(x3; y3; z3) не лежат на одной прямой, то проходящая через них плоскость представляется уравнением:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | x-x1 | y-y1 | z-z1 | | x2-x1 | y2-y1 | z2-z1 | | x3-x1 | y3-y1 | z3-z1 | |  | | = 0 |

Уравнение плоскости A1A2A3

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | |  |  |  | | --- | --- | --- | | x+5 | y-0 | z-1 | | 1 | -2 | 2 | | 11 | 2 | 10 | |  | | = 0 |

(x+5)((-2) • 10-2 • 2) - (y-0)(1 • 10-11 • 2) + (z-1)(1 • 2-11 • (-2)) = -24x + 12y + 24z-144 = 0  
Упростим выражение: -2x + y + 2z-12 = 0